Short Course on Chemofluidics
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Short Course on Chemofluidics – Part 1

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Course Outline

A/Part 1. Intro
• Basic Pneumatic Components
• Pneumatic Modeling
• Chemofluidic Configurations
  • Centralized
  • Direct Injection
  • Hybrid
• Chemofluidic Components and Materials

B. Pneumatic Control
2. Impedance Control
3. Nonlinear Controller with Pressure Observer
4. Control using on/off switching valves

C/Part 5. Chemofluidic Valve Designs
• Hot Gas Servovalve
• Proportional Injection Valve

D. Chemofluidic Modeling
6. Energetic Approach

E. Chemofluidic Control
7. Linearized predictive control with solenoid injectors in DI configuration
8. Nonlinear control of proportional injection DI configuration
Pneumatic Components: Actuators

Cylinders

- Single acting
- Double acting
- Single rod
- Double rod
- Seals: cup seals, o-ring seals

Rotary Actuators

- Not full revolution

Vane Motors

- Full rotation

Bimba
Numatics
Festo
SMC
Norgren
Clippard
Others …
Pneumatics Components: Valves

Control Valve Schematics

Types:

2-way
- Solenoid
- Solenoid/piloted

3-way
- 2 position: charge/exhaust
- 3 position: charge/closed/exhaust
Pneumatics Components: Valves (cont)

4-way

- 2-position: A: charge/exhaust, B: exhaust/charge
- 3-position: charge/closed/exhaust, charge/open/exhaust
- proportional
Pneumatics Components: Valve Manufacturers

**Parker**
- Series 9: fast (<10ms), small orifice, liquid/gas 2 and 3-way solenoid valves

**Festo**
- High bandwidth (<100 Hz) 4-way proportional valves

**Enfield Technologies**
- Small, high bandwidth 4-way proportional valves

**Numatics**
- Small, fast, inexpensive 4-way 2 and 3-position solenoid valves
Pneumatics Components: Pressure Sensors

Omega
Festo
Others …

Output types:
- Current
- Voltage
Pneumatics – Control Volume

Energetic Analysis

CV

• Internal energy (storage)
• Work (flux)
• Heat (flux)
• Enthalpy (flux) $h=u+pv$, what does this mean?
• KE flow (~0)
• Gravitational PE ~0

Constitutive Equation

• $U_{dot} = H_{dot} + Q_{dot} - W_{dot}$

Sign Conventions

Effort/Flow pairs
Aux: \[ C_V = \frac{R}{\gamma - 1} \quad \delta = \frac{C_p}{C_v} \quad PV = mRT \quad \text{Assume perfect gas} \]

1\textsuperscript{st} law:

\[
\dot{U} = \dot{H} + \dot{Q} - \dot{W} \]

charge = sum of fluxes inside \text{ in and out}

Internal energy of a gas (definition):

\[ \Delta U = mc_v \Delta T \]

depend only on temp

only know the change, not the total quantity.

\[
dU = mc_v dT,
\]

\[
\int_{u_{\text{ref}}}^{u} dU = \int_{T_{\text{ref}}}^{T} mc_v dT
\]

\[
U - U_{\text{ref}} = mc_v (T - T_{\text{ref}})
\]

\[
\dot{U} - \dot{U}_{\text{ref}} = mc_v \dot{T} + mc_v \dot{T}_{\text{ref}} - mc_v \dot{T}_{\text{ref}}
\]

\[
\dot{U} = mc_v \dot{T} + mc_v \dot{T}_{\text{ref}}
\]

- rate of change of temp inside \( CV \)
- total mass inside \( CV \)
- temp inside \( CV \)
- mass entering (+) or leaving (-) \( CV \)
- rate of change of internal energy stored inside \( CV \)
All expressions for $\dot{U}$:

Given: $\dot{U} = \frac{d}{dt} (mc_vT)$

$C_v = \frac{R}{\gamma - 1}$

$\dot{U} = \frac{d}{dt} \left( \frac{mRT}{\gamma - 1} \right)$ \quad PV = mRT

$\dot{U} = \frac{d}{dt} \left( \frac{PV}{\gamma - 1} \right)$

$\dot{U} = \frac{PV + P \dot{V}}{\gamma - 1}$

**Enthalpy**

$h = u + pv$ \quad (mass specific)

all terms \quad \left[ \frac{\text{joules}}{\text{kg}} \right]

$H = U + PV$


$H \rightarrow CV$

$CV = CV$

work$

Total energy pushed in = dU + work
Total energy in = $dH_{in}$

$dH_{in} = dU + Work$

$dU = dm_{in}c_vT_{in}$

$Work = PdV$

Use $PV = nRT$:

$PdV = dm_{in}RT_{in}$

$dH_{in} = dU + Work$

$dH_{in} = dm_{in}c_vT_{in} + dm_{in}RT_{in}$

$= dm_{in}(c_v + R)T_{in}$

$\overline{c_p} = c_p$

$\frac{dH}{dt} = \frac{dm_{in}c_pT_{in}}{dt}$

$\dot{H}_{in} = \dot{m}_{in}c_pT_{in}$

Note: $\dot{H}_{out} = \dot{m}_{out}c_pT_{out}$

$\Rightarrow \dot{H} = \dot{m}c_pT_{flow}$

$m > 0$ means flow in, $T_{flow} = T_{in}$

$m < 0$ means flow out, $T_{flow} = T$
\[ U = H + Q - W \]

**Heat flow rate:**

\[ Q = 0 \quad \text{typically assumed for mechanics} \]

*Note: this is not the case for real fluids (in the real world)*

\[ Q \] will be an amount of modeling the heat of reaction

**Work rate:**

\[ W \] positive for work done by the CV

\[ \text{def: } dW = Fd\dot{x} \]

\[ \text{piston: } \quad \frac{d\dot{x}}{A} \]

\[ F = pA \]

\[ dv = A d\dot{x} \implies d\dot{x} = \frac{dv}{A} \]

\[ dW = Fd\dot{x} \]

\[ = (pA) \left( \frac{dv}{A} \right) \]

\[ dW = pdv \]

\[ \dot{W} = pdv \]

*Note: \( \dot{W} \neq p\dot{V} + p\dot{V} \)*
Total

\[ \dot{U} = mc_v T + mc_p T \]
\[ \dot{U} = \frac{\dot{P} V + \dot{P} \dot{V}}{\gamma-1} \]

\[ \dot{H} = mc_p T_{\text{flow}} \]
\[ Q = 0 \quad (\text{adiabatic}) \]
\[ W = \dot{P} V \]

Two useful expressions

I. \[ \frac{\dot{P} V + \dot{P} \dot{V}}{\gamma-1} = mc_p T_{\text{flow}} - \dot{P} \dot{V} \]

\[ \dot{P} V + \dot{P} \dot{V} = m (\gamma-1) c_p T_{\text{flow}} - (\gamma-1) \dot{P} \dot{V} \]

\[ \uparrow \]

\[ -\dot{P} \dot{V} + \dot{P} V \]

\[ \dot{P} V = m (\gamma-1) c_p T_{\text{flow}} - \dot{P} \dot{V} \]

Use \[ c_v = \frac{R}{\gamma-1} \]
\[ s = \frac{c_p}{c_v} \]
\[ \downarrow \]
\[ c_v = \frac{c_p}{s} \]
\[ \frac{c_p}{s} = \frac{R}{\gamma-1} \]
\[ (\gamma-1) c_p = \dot{P} \dot{V} \]

Adiabatic

\[ \dot{P} = \frac{\dot{P} m R T_{\text{flow}} - \dot{P} \dot{V}}{V} \]

We'll see this in nearly everything.

You influence \( m \) with valves. \( \dot{P} \) evolves due to both

Environment influences \( V, \dot{V} \) to both
The other expression

\[ mc_v T + mc_v \dot{T} = mc_v T_{\text{flow}} - PV \]

Solve for \( \dot{T} \)

Tells you how temp in CV evolves

- as m of temp \( T_{\text{flow}} \) enters or leaves the CV
- as work is done by or on CV influences by environment

\[ \dot{Q} \text{ takes on whatever it needs to keep } T_{cv} \text{ const. so } \dot{U} = H + Q - W \] with no assumptions on \( \dot{Q} \), we still know\[ \dot{U} = \frac{d}{dt} (mc_v T) = \frac{\dot{P}V + P\dot{V}}{\gamma - 1} \] (only assumed ideal gas.)

\[ mc_v \dot{T} + mc_v \ddot{T} = \frac{\dot{P}V + P\dot{V}}{\gamma - 1} \]

\[ \dot{T} \text{ of what? } T_{cv} \]

\[ m(\gamma - 1)cv T = \dot{P}V + P\dot{V} \]

\[ \dot{R} \]

Note this is no longer \( T_{\text{flow}} \) ! it is \( T \) of CV

\[ \dot{P} = \frac{\dot{m}RT}{V} - \frac{P\dot{V}}{V} \]

Only different by \( \gamma \)

\[ \dot{P} = \frac{\gamma \dot{m}RT_{\text{flow}} - \dot{P}\dot{V}}{V} \]

\( \gamma \) is \( 1.4 \) for air.
Pneumatics – mass flow

Derived from isentropic flow of an ideal gas through a converging nozzle (see Fox and McDonald text, Richer and Hurmuzlu paper)

Choked
- $P_{up} > \frac{P_{down}}{P_{crit}} = \frac{P_{down}}{0.528} \sim 2 \ P_{down}$
- $P_{up} > \sim 2 \ P_{down}$

Unchoked
- $P_{up} < \sim 2 \ P_{down}$

Variables:
- Upstream pressure
- Downstream pressure
- (Stagnation) Temperature of flow
- Flow orifice area

Parameters:
- Discharge coefficient $C_f$ (between 0 and 1)

Mathematical expression:

$$\dot{m}_v = \begin{cases} 
C_f A_v C_1 \frac{P_u}{\sqrt{T}} & \text{if } \frac{P_d}{P_u} \leq P_{cr} \\
C_f A_v C_2 \left( \frac{P_d}{P_u} \right)^{1/k} \sqrt{1 - \left( \frac{P_d}{P_u} \right)^{(k-1)/k}} & \text{if } \frac{P_d}{P_u} > P_{cr}
\end{cases}$$

$$C_1 = \sqrt{\frac{k}{R}} \left( \frac{2}{k+1} \right)^{1/k+1} \quad ; \quad C_2 = \sqrt{\frac{2k}{R(k-1)}}$$

$$P_{cr} = \left( \frac{2}{k+1} \right)^{k/k-1}$$
Pneumatics – mass flow

Constants (Air, room temp)

- $C_1 = 0.0404$, $C_2 = 0.1562$
- $P_{cr} = 0.528$, $k = 1.4$, $C_f = 1$
- Orifice diameter = 3mm

\[
\dot{m}_v = \begin{cases} 
C_f A_v C_1 \frac{P_u}{\sqrt{T}} & \text{if } \frac{P_d}{P_u} \leq P_{cr} \\
C_f A_v C_2 \left( \frac{P_u}{P_d} \right)^{1/k} \sqrt{1 - \left( \frac{P_d}{P_u} \right)^{(k-1)/k}} & \text{if } \frac{P_d}{P_u} > P_{cr}
\end{cases}
\]

\[
C_1 = \sqrt{\frac{k}{R}} \left( \frac{2}{k+1} \right)^{k+1/k-1} \\
C_2 = \sqrt{\frac{2k}{R(k-1)}} \\
P_{cr} = \left( \frac{2}{k+1} \right)^{k/k-1}
\]
Pneumatics – mass flow

Mass Flow Rate Equation Notes:

• Only results in a real value for \( P_{\text{up}} = P_{\text{down}} \).
• \( \dot{m} \) always \( \geq 0 \).
• You must switch \( P_{\text{up}} \) and \( P_{\text{down}} \) depending on the condition across the valve (charge or exhaust).
• You need to multiply \( \dot{m} \) by -1 if the flow is out of the CV to agree with the sign convention of the pressure dynamics equation.
• For a given \( P_{\text{up}} \) and \( P_{\text{down}} \), there is a one-to-one algebraic relationship between mass flow and \( A_v \). This will tell you what to set the valve at to get a desired mass flow rate!

Proportional Valves and Orifice Size \( (A_v) \)

• In the mass flow rate equation, you set \( A_v \)

Valve specifications: \( C_v \) \((K_v \text{ metric}) \) or \( A_v \)

• \( C_v \) from hydraulic flow equation
• Relationship between \( A_v \) and \( C_v \)
• Proportional valves will indicate maximum \( A_v \) or \( C_v \)

Valve specs:

• Proportional: Bandwidth, power consumption
• On/off: opening/closing time
Chemofluidic Systems

propellant + catalyst = reaction products + heat

- Hydrogen peroxide decomposes into a hot gas which can be used to power a pneumatic actuator
Chemofluidic Configurations: Centralized

- Pressurized inert gas
- Liquid monopropellant
- Propellant line
- Liquid propellant valve
- Pressure control loop
- Hot gas line
- Hot gas reservoir
- Catalyst pack
- 4-way proportional valve
- Actuator output shaft
- Gas actuator
- VC SOL
Chemofluidic Configurations: Direct Injection (Solenoid)
Chemofluidic Configurations: Direct Injection (Proportional)

Chemofluidic configurations involve the integration of chemical reactions with fluid dynamics. In this diagram, the configuration includes a fuel tank, monopropellant, and an inert gas charge. The fuel line connects to an actuator, and there is a catalyst pack. The exhaust valve (gas) and inlet valve (liquid) are also part of the system. The diagram illustrates the flow and control mechanisms in a chemofluidic setup.
Chemofluidic Configurations: Hybrid

- Pressurized inert gas
- Liquid monopropellant
- Propellant line
- Liquid propellant valve
- Pressure control loop
- Hot gas line
- Hot gas reservoir
- Catalyst pack
- Actuator output shaft
- Gas actuator
Chemofluidic Components and Materials

Hydrogen Peroxide
- FMC
  - One gallon “sample”
  - 70% HTP (Rocket Grade)
  - ~$350 (mostly shipping)
- Degussa-Hüls
  - 55 gallon drum – inexpensive
  - Stabilizers poison the catalyst

Catalyst
- Shell 405
  - Iridium coating on alumina pellets
  - 14-18 mesh
  - ~$10,000 / lb
  - Sources: General Dynamics, Honeywell (contact Vanderbilt for names)
- Catalyst-Pack
  - Stainless Steel tubing (1/4”, 1/8”, 1/16” diameter and 1” to 3” long) with screens and fittings on both ends.
Chemofluidic Components and Materials (cont)

Cylinders
- Must have viton seals (not buna-N)
- Bimba original-line stainless steel cylinders have this option

Valves
- For on/off H2O2 flow: Parker Series 9 valves
- Hot gas valves: custom proportional (contact Vanderbilt)

Compatible Materials
- Stainless Steel (318 ok, not 304)
- Aluminum (5254, 6061)
- Others, see chart

Flexible Fuel Lines
- Upchurch scientific – PEEK tubing 1/8 x 0.08, use A-Lok fittings
- DH400 flexible tube F1818NF-12 P/N:400718, www.dhinstruments.com

Hot Gas Lines
- Stainless steel tubing (1/4”, 1/8”, 1/16”) and A-loc fittings
- PEEK tubing (for 70% HTP or less – temp limited)
Short Course on Chemofluidics – Part 2

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IMPEDANCE CONTROL OF A PNEUMATIC ACTUATOR FOR CONTACT TASKS

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Motivation

Problem
Compliant manipulation, as in assembly tasks, requires the manipulator to have accurate position tracking and soft collision while making contact with an uncertain environment. Typical industrial robot actuators (hydraulic and motor/gearhead systems) are fundamentally admittances possessing high stiffness. By utilizing force measurements, a lot of approaches have been done to transform the admittance into an impedance, but all have their tradeoffs.

General Objective
Design a method for the impedance control of a pneumatic linear actuator for tasks involving contact interaction by taking advantage of the natural compliance of pneumatic actuators without using a load cell.

Approach
The control methodology presented contains an inner loop to control the pressure on two sides of a pneumatic cylinder, while an outer loop enforces an impedance relationship between external forces and motion and commands desired pressures to the inner loop.
The pressure in each side of the actuator is separately controlled with a three-way proportional valve.

The inner loop enforces the natural compliance of the pneumatic actuator by controlling both the sum and difference of the pressures on both sides of the pneumatic actuator.

This is accomplished by utilizing two three-way proportional spool valves instead of a four-way valve typically used in fluid power control.
Pressure Controller

The rate of change of pressure within each pneumatic chamber can be expressed as:

\[ \dot{P} = \frac{rRT}{V} \dot{m} - \frac{rPV}{V} \]

Defining:

\[ b(x) = \frac{rRT}{V} > 0 \quad f(x) = -\frac{rPV}{V} \quad x^T = [V \quad \dot{V} \quad P] \]

The pressure dynamics can be stated more conveniently as:

\[ \dot{P} = f(x) + b(x)u \]

Sliding mode control can be utilized to establish pressure tracking control

\[ s = e + 2\lambda \int e + \lambda^2 \int \int e \quad e = P - P_d \]

Defining the standard positive-definite Lyapunov function:

\[ V = \frac{1}{2}s^2 \]

The derivative of the Lyapunov function is enforced to be:

\[ \dot{V} = s\dot{s} = -K|s|b(x) \leq 0 \]

Solving for \( \dot{s} \) gives:

\[ \dot{s} = -K \text{sgn}(s)b(x) \quad K > 0 \]

Taking derivative of \( s \) gives:

\[ \dot{s} = b(x)u + f(x) - \dot{P}_d + 2\lambda e + \lambda^2 \int e \]

Typical sliding mode control law can be derived as:

\[ u = \frac{1}{b(x)} \left[ \dot{P}_d - f(x) - 2\lambda e - \lambda^2 \int e \right] - K \text{sgn}(s) \]
Pressure Controller (con’t)

Mass flow rate can be expressed as function of upstream and downstream pressure:

\[ \dot{m} = A\Psi(P_u, P_d) \]

For charging and discharging case, the area normalized mass flow rate can be written as:

\[
\Psi(P_u, P_d) = \begin{cases} 
\Psi(P_s, P) & \text{for } A \geq 0 \\
\Psi(P, P_{atm}) & \text{for } A < 0 
\end{cases}
\]

A common mass flow rate model used for compressible gas flowing through a valve:

\[
\Psi(P_u, P_d) = \begin{cases} 
\frac{C_1 C_f P_u}{\sqrt{T}} & \text{if } \frac{P_d}{P_u} \leq C_r \text{ (choked)} \\
\frac{C_2 C_f P_u}{\sqrt{T}} \left( \frac{P_d}{P_u} \right)^{1/k} \sqrt{1 - \left( \frac{P_d}{P_u} \right)^{(k-1)/k}} & \text{otherwise (unchoked)} 
\end{cases}
\]

So, the required valve area can be expressed as:

\[
A = \begin{cases} 
u / \Psi(P_s, P) & \text{for } u \geq 0 \\
u / \Psi(P, P_{atm}) & \text{for } u < 0
\end{cases}
\]
Impedance Control

System dynamics: \[ M\ddot{x} + B\dot{x} + F_f = F + F_e \]

where \( F = P_a A_a - P_b A_b - P_{atm} A_r \) is the pneumatic actuation force.
\( F_e \) is the force acted by the environment.

The desired dynamic impedance behaviour: \[ m(\ddot{x} - \ddot{x}_d) + b(\dot{x} - \dot{x}_d) + k(x - x_d) = F_e \]

The approach taken here is to avoid using a load cell in favour of utilizing the acceleration. To cancel the original system dynamics and enforce this impedance behaviour, the desired actuation force is therefore required to be:

\[ F_d = (\tilde{M} - m)\ddot{x} + (\tilde{B} - b)\dot{x} - k(x - x_d) + m\ddot{x}_d + b\dot{x}_d + \tilde{F}_f \]

\( \tilde{M}, \tilde{B} \) and \( \tilde{F}_f \) are the estimated inertia, damping and coulomb friction of the pneumatic actuator.

Assuming that the actual force \( F \) can be driven to the desired force \( F_d \) through rapid and accurate pressure control, the dynamics become:

\[ m(\ddot{x} - \ddot{x}_d) + b(\dot{x} - \dot{x}_d) + k(x - x_d) = F_e + (\tilde{M} - M)\ddot{x} + (\tilde{B} - B)\dot{x} + (\tilde{F}_f - F_f) \]

Which is a close approximation of the desired impedance behaviour.
Impedance Control

Specify the compliance of the actuator using pressure sum in two chambers:

\[ P_{ad} + P_{bd} = P_{sum} \quad P_{ad} A_a - P_{bd} A_b - P_{atm} A_r = F_d \]

Solving the two equations gives the desired pressure in both chambers:

\[ P_{ad} = \frac{F_d + P_{sum} A_b + P_{atm} A_r}{A_a + A_b} \quad P_{bd} = P_{sum} - P_{ad} \]

Coulomb friction is modelled as:

\[ \tilde{F}_f = -F_c \text{ sgn}(\dot{x}_d) \]

\( F_c \) is defined asymmetrically as:

\[ F_c = \begin{cases} F_{cpos} (\dot{x}_d > 0) \\ F_{cneg} (\dot{x}_d < 0) \end{cases} \]

Friction model plays an important role in position tracking.
Experimental Setup

Experiments were conducted to show:

- The pressure tracking performance.
- The motion control of the actuator in free-space.
- The force transition from non-contact to contact when hitting an unpredicted stiff wall.
Pressure Tracking and Motion Tracking in Free Space

**Pressure Tracking**

- **1Hz**
- **6Hz**
- **10Hz**

**Motion Tracking**

- **0.25Hz**
- **1Hz**
- **1.5Hz**
Non-contact to Contact Transition

\[ m = 0.5 \text{ kg} \quad b = 200 \text{ N/(m/s)} \quad k = 800 \text{ N/m}. \]

\[ m = 1 \text{ kg} \quad b = 400 \text{ N/(m/s)} \quad k = 1600 \text{ N/m}. \]

\[ m = 2 \text{ kg} \quad b = 800 \text{ N/(m/s)} \quad k = 3200 \text{ N/m}. \]

The ramp is commanded to stop at 60mm, the stiff wall is at about 55.6mm.

Each set of parameters has the same damping ratio \( \xi = 5 \) and natural frequency \( \omega_n = 40 \text{ rad/sec} \). \( P_{sum} = 400 \text{ Kpa} \) for all three cases.
Conclusions

• A method for the impedance control of a pneumatic linear actuator for tasks involving contact interaction was presented.

• By exploiting the natural compliance properties of a pneumatic actuator, the impedance control method presented does not require the use of a load cell to measure the interaction force, but rather allows the use of a low-bandwidth acceleration feedback signal instead.

• A controller to achieve desired pressure tracking in each side of the pneumatic cylinder was also presented.

• Experimental results show good pressure tracking, good motion tracking in free-space, and a predictable trend of lower contact forces for lower target inertias of the system.
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Short Course on Chemofluidics – Part 3

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Pressure Observer Based Servo Control of Pneumatic Actuators

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Presentation Outline

• Motivation

• Dynamics of Pneumatic Actuators

• Literature Survey

• Pressure Observers

• Controller Design

• Experiments

• Results

• Conclusions
Motivation

• Pneumatic systems use is limited despite its many advantages such as reliability, high power to weight ratio, and compliance for interaction tasks.

• High cost of the actuators mainly because of pressure sensors.

• Pressure sensors are bulky and large.

• Typical Cost: $250 to $500 per sensor

• System requires two pressure sensors per degree of freedom. Adds 30% percent to the initial costs.

• Use of pressure observers allows model-based control without direct pressure sensing.
Dynamics of Pneumatic Actuators

LOAD DYNAMICS

\[ M \ddot{x} + B \dot{x} + F_c = P_a A_a - P_b A_b - P_{atm} A_r \]

PNEUMATIC CYLINDER DYNAMICS

\[ \dot{p}_{(a,b)} = \frac{\gamma RT}{V_{(a,b)}} \dot{m}_{(a,b)} - \frac{\gamma \dot{V}_{(a,b)}}{V_{(a,b)}} P_{(a,b)} \]

VALVE DYNAMICS

\[ \dot{m}_e = \frac{C_e A_e P_u C_2}{\sqrt{T_e}} \left( \frac{P_d}{P_u} \right)^{\gamma k} \sqrt{1 - \left( \frac{P_d}{P_u} \right)^{\gamma - 1}} \]

if \[ \frac{P_d}{P_u} > \left( \frac{2}{\gamma + 1} \right)^{\gamma - 1} \] unchoked flow

\[ \dot{m}_e = \frac{C_e A_e P_u C_1}{\sqrt{T_e}} \]

if \[ \frac{P_d}{P_u} \leq \left( \frac{2}{\gamma + 1} \right)^{\gamma - 1} \] choked flow
Literature Survey

- Pandian et. al. [1], Richer and Hurmuzlu [2], and many others [3,4] presented the design of a controller that can provide stable position/force tracking for high bandwidths (up to 20 Hertz). These controllers ignored energetic efficiency and/or initial costs associated with the system.

- Sanville et al. [5], Al-Dakkan et al.[6], and others presented control methodologies that provide significant energy savings without sacrificing the tracking accuracy.

- Ye et al. [7], Kunt and Singh [8], Lai et al. [9], Royston and Singh [10], Paul et al. [11], and Shih and Hwang [12] demonstrated the viability of servo-control of pneumatic actuators via solenoid on/off valves in place of proportional valves.

- Gulati and Barth [13] presented the design of two non-linear pressure observers and also discussed the stability, robustness, and convergence.
Pressure Observers

Force-Error Based Lyapunov Observer Design

Estimating pressure in chambers by the following equation

\[ \dot{\hat{P}}_a = \frac{\gamma RT}{V_a} \dot{\hat{m}}_a - \frac{\gamma \dot{V}_a}{V_a} \hat{P}_a + k_1 \Delta \tilde{F} \quad \text{and} \quad \dot{\hat{P}}_b = \frac{\gamma RT}{V_b} \dot{\hat{m}}_b - \frac{\gamma \dot{V}_b}{V_b} \hat{P}_b + k_2 \Delta \tilde{F} \]

where \( \Delta \tilde{F} = (P_a A_a - P_b A_b) - (\hat{P}_a A_a - \hat{P}_b A_b) \)

Rearranging \( \Delta \tilde{F} = (P_a - \hat{P}_a) A_a - (P_b - \hat{P}_b) A_b \)

Actual difference of the force between the two sides of the piston is measured by the state equation:

\[ P_a A_a - P_b A_b = M \ddot{x} + B \dot{x} + F_c - P_{\text{atm}} A_r \]

Energy-Based Lyapunov Observer Design

Estimating pressure in chambers by the following equation

\[ \dot{\hat{P}}_a = \frac{RT}{V_a} \dot{\hat{m}}_a - \frac{\dot{V}_a}{V_a} \hat{P}_a \]

\[ \dot{\hat{P}}_b = \frac{RT}{V_b} \dot{\hat{m}}_b - \frac{\dot{V}_b}{V_b} \hat{P}_b \]
Pressure Observers

Actual/ Observed Pressure with Energy-Based Observer at 3 Hz – Chamber ‘A’

Actual/ Observed Pressure with Energy-Based Observer at 2 Hz – Chamber ‘A’
Design of a Sliding Mode Controller

Choosing the positive define Lyapunov scalar function as:

\[ V = \frac{1}{2} s^2 \]

Select control input such that \( \dot{V} = s \dot{s} < 0 \) (Lyapunov Stability)

Define a time-varying sliding surface as:

\[ s = \left( \frac{d}{dt} + \dot{\lambda} \right)^{n-1} e \]

\[ s = (\ddot{x} - \ddot{x}_d) + 2 \lambda \ddot{e} + \lambda^2 e \]

Differentiating and substitution results in:

\[ \dot{s} = \frac{1}{M} (\dot{P}_a A_a - \dot{P}_b A_b - B \ddot{x}) - x_d^{(3)} + 2 \lambda \ddot{e} + \lambda^2 \dot{e} \]

Four-way proportional spool valve constraint:

\[ A_v = A_{v_a} = -A_{v_b} \]
Design of a Sliding Mode Controller

Solving for the control input yields:

\[
A_{\text{veq}} = \frac{\gamma \dot{x} \left( \frac{P_a A_a^2}{V_a} + \frac{P_b A_b^2}{V_b} \right) + B \ddot{x} + M \left( x_d^{(3)} - 2 \lambda \ddot{e} - \lambda^2 \dot{e} \right)}{RT\gamma \left( \frac{C_f}{\sqrt{T}} \right) \left( \frac{P_{u_a} \psi(P_{u_a}, P_{d_a}) A_a}{V_a} + \frac{P_{u_b} \psi(P_{u_b}, P_{d_b}) A_b}{V_b} \right)}
\]

where,

\[
\psi = \begin{cases} 
P_u \gamma \left( \frac{2}{\gamma + 1} \right)^{(\gamma+1)\gamma - 1)} & \text{for choked flow} \\
P_u \left( \frac{P_d}{P_u} \right)^{\frac{1}{\gamma}} \left( \frac{2\gamma}{R(\gamma - 1)} \right) & \text{for unchoked flow}
\end{cases}
\]

A robustness term is added to this control input to provide uniform asymptotic stability

\[
A_v = A_{\text{veq}} - k * \text{sat}(\frac{s}{\phi})
\]

\[
\text{sat}(\frac{s}{\phi}) = \begin{cases} 
\text{sgn}(\frac{s}{\phi}) & \text{if } \text{abs}(\frac{s}{\phi}) \geq 1 \\
\frac{s}{\phi} & \text{otherwise}
\end{cases}
\]
Experimental Setup

Experimental Setup of Pneumatic Actuator Servo System
Observer-Based Sliding Mode Controller (1)

**Desired/ Actual Position at 0.25 Hz Frequency using Pressure Sensors**

**Desired/ Actual Position at 0.25 Hz Frequency using Pressure Observers**
Observer-Based Sliding Mode Controller (2)
Observer-Based Sliding Mode Controller (3)
Observer-Based Sliding Mode Controller (4)
Conclusions

• The design of a controller that uses pressure observers is presented.

• The results presented demonstrate that the tracking performance using pressure observers versus using pressure sensors is in essence indistinguishable.

• Since the system can be controlled accurately using pressure observers, it results in a low cost system (Savings: 30 percent in initial costs) with no trade-off in the performance of the controller.

• Additionally, the use of pressure observers along with the controller developed results in lower weight, more compact, and lower maintenance system.
References


References


Short Course on Chemofluidics – Part 4

Eric J. Barth
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On/Off Valves for Servo-Control

Idea: Use inexpensive on/off solenoid valves to accomplish what a proportional servo valve accomplishes.
On/Off Valves for Servo-Control
Direct Switching Approach

• Plant dynamics:

\[
\frac{d^n}{dt^n} x = f(x) + b(x)u
\]

with state vector, \( x^T = [x, \dot{x}, \ldots, x^{(n-1)}] \)

• The input to the plant is only allowed to be a finite set of \( p \) discrete values:

\[ u \in \{u_1, u_2, \ldots, u_p\} \]

For the pneumatic servo problem, these correspond to the four combinations of:

<table>
<thead>
<tr>
<th>Valve 1</th>
<th>Valve 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) pressurize</td>
<td>exhaust</td>
</tr>
<tr>
<td>2) exhaust</td>
<td>exhaust</td>
</tr>
<tr>
<td>3) exhaust</td>
<td>pressurize</td>
</tr>
<tr>
<td>4) pressurize</td>
<td>pressurize</td>
</tr>
</tbody>
</table>
Nonlinear Control Design:

• Define the continuous positive definite Lyapunov function:

\[ V = \frac{1}{2} s^2 \]

• Choose the typical integral sliding surface:

\[ s = \left( \frac{d}{dt} + \lambda \right)^n \int e dt = \lambda^n \int e dt + \sum_{r=0}^{n-1} \binom{n}{r} \lambda^r \frac{d^{n-r-1}}{dt^{n-r-1}} e \]

where \( e = x - x_d \)

• Note that \( s \) is a measurable quantity if,

\[ x_d^{(i)} \text{ for } i = \{0, 1, \ldots, n\} \text{ is known and } x^{(i)} \text{ for } i = \{0, 1, \ldots, n-1\} \text{ is measurable (i.e. full state feedback)} \]
On/Off Valves for Servo-Control
Direct Switching Approach

Nonlinear Control Design (cont.):

• Stability guaranteed and error driven to zero when:

\[ \dot{V} = s\dot{s} \leq 0 \]

• Quantity \( s \) is measurable.

• Quantity \( \dot{s} \) is given by:

\[
\dot{s} = b(x)u + f(x) - \frac{d^n}{dt^n} x_d + \sum_{r=1}^{n} \binom{n}{r} \lambda^r \frac{d^{n-r}}{dt^{n-r}} e
\]

Non-candidate term is calculable from known and measured values.
Candidate term depends upon selection of \( u \in \{ u_1, u_2, \ldots, u_p \} \)
Nonlinear Control Design (cont.):

- Each input value $u_i$ will have an associated $\dot{V}_i$ term:

$$\dot{V}_i = s(e^{(n-1)}, \ldots, e) \dot{s}(u_i, x, x_d^{(n)}, e^{(n-1)}, \ldots, e)$$

for $i = \{1, 2, \ldots, p\}$

- Stability and error convergence criterion $\dot{V} = ss \leq 0$ indicates which $u_i$ to select.

Control law:

$$u = u_i \text{ such that } \dot{V}_i = \max_{j=\{1, 2, \ldots, p\}} (\dot{V}_j) \text{ and } \dot{V}_i \leq 0$$

for $i = \{1, 2, \ldots, p\}$
Nonlinear Control Design (cont.):

Control law:

\[ u = u_i \quad \text{such that} \quad \dot{V}_i = \max_{j=1,2,...,p} (\dot{V}_j) \quad \text{and} \quad \dot{V}_i \leq 0 \]

for \( i = \{1,2,\ldots, p\} \)

- Selects least negative \( \dot{V}_i \)
- Directly enforces stability and error convergence condition
- Method applicable to more general systems (i.e. \( x^{(n)} = f(x,u) \)) since control variable is not being “solved for”.
- Posses problems for systems of high order where derivatives of \( x \) are not measurable and must be obtained through differentiation (noise problems).
Incorporating valve time-delay:

- **Plant with time delay** $T_D$:
  \[
  \frac{d^n}{dt^n} x(t) = f_i(x(t), u(t - T_D)), \quad \forall u \in \{u_i : i = 1, 2, \ldots, p\}
  \]
- **Present control** $u(t)$ takes effect at time $t + T_D$
  \[
  \hat{V}_i(t + T_D) = \hat{s}(t + T_D) \hat{s}(t, t + T_D)
  \]
- **Need a model-based prediction of** $\hat{x}(t + T_D)$
  and have $x_d(t + T_D)$ available. Use a predictor.
On/Off Valves for Servo-Control
Direct Switching Approach

- 85 psig supply pressure
- Two 2-position 3-way solenoid valves
- 10 kg mass on linear bearings
- Linear potentiometer (for feedback control)
- Three pressure sensors (for modeling only!)
On/Off Valves for Servo-Control
Direct Switching Approach

0.5 Hz position tracking of end mass
On/Off Valves for Servo-Control
Direct Switching Approach

1.0 Hz position tracking of end mass
On/Off Valves for Servo-Control
Direct Switching Approach

2.0 Hz position tracking of end mass
3.0 Hz position tracking of end mass
On/Off Valves for Servo-Control
Direct Switching Approach

A method was presented for the control design of switching systems. This methodology was applied to a pneumatic positioning system.

- No pressure sensors (needs model simplification in each mode – method allows for this)
- Implements inexpensive solenoid-type binary valves
- Stability robustness and performance addressed directly with a Lyapunov approach
- Time delay associated with valves incorporated into control design
- Applicable to the general class of non-autonomous nonlinear systems that are not affine in the control variable
- Experimental pneumatic position tracking control shown
- Alleviates unneeded switching symptomatic of PWM approaches
Short Course on Chemofluidics – Part 5

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Design and Characterization of a Rotary Actuated Hot Gas Servovalve

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Producing Energy with a Monopropellant

\[ \text{propellant} + \text{catalyst} = \text{reaction products} + \text{heat} \]
Proportional Injector DI System

- inert gas charge
- fuel tank
- monopropellant
- fuel line
- catalyst pack
- exhaust valve (gas)
- actuator
- inlet valve (liquid)
Exhaust Valve Design Considerations

• **Accommodate high temperature steam** *(232°C)*
  - Spool/sleeve design must ensure uniform thermal expansion
  - Design must incorporate sufficient thermal isolation between the actuator and spool/sleeve/manifold

• **Incorporate a rotary actuated spool**
  - Enables the use of rotary electromagnetic actuators (i.e., DC motors)
  - Actuator and sensor integrated into a single compact package
  - Provides for the incorporation of a transmission in order to obtain the torque/speed characteristics required by the spool
  - Position control is straightforward and simple to implement
Three-Way Design

- Sleeve is suspended in manifold with o-rings to isolate respective flow paths.
- Sleeve incorporates radial grooves and two opposing holes for fluid flow.
- As such, the spool operates in a pressure-balanced environment, minimizing friction and creating an air bearing effect on the spool.
Three-Way Design (cont.)

- Spool has two orthogonal holes that connect to a central exhaust port running axially along the spool.
- Holes are aligned such that either port A or port B is connected to exhaust, with the other port blocked.
- Spool rotation of 90° encompasses the full range of 3-way valve operation.
Four-Way Design

- Sleeve has four radial grooves, with four holes each for supply/exhaust and two holes each for ports A/B
- Spool has two sets of opposing slots, oriented 90° apart to connect chambers A and B alternately to supply/exhaust
- In the schematic above, chamber A is connected to supply and chamber B is connected to exhaust
Four-Way Design (cont.)

- In this configuration, chamber A is connected to exhaust and chamber B is connected to supply
Four-Way Design (cont.)

- In this configuration, both chambers A and B are blocked
Three-Way Hot Gas Prototype

- Incorporates thermal isolation for the DC servomotor (max. operating temperature of 85°C)
- Thermal isolation provided by two pieces fabricated from PEEK (polyetherether-ketone)
  - motor mount provides thermal standoff from the manifold
  - bearing element of Oldham coupling provides thermal standoff from the spool
Three-Way Hot Gas Prototype (cont.)

- Spool/sleeve have high tolerance fit, are designed from the same material, and are designed to have low residual stresses from fabrication.

- Spool/sleeve fabricated from 440 stainless steel, which was hardened, ground, and polished to attain ~2.5 micron diametral clearance.

- O-ring isolation of the spool/sleeve from the remainder of the valve body avoids any thermally induced interference that would otherwise result in seizure.
Characteristics of Exhaust Valve Prototype

• Control of $C_v$ between 0 and 0.45 (with resolution of 0.002): 
  \[ C_v = Q \sqrt{\frac{\gamma_w}{P_{up} - P_{down}}} \]

• Tested to pressures of 300 psi (pressure limited to 300 psi by other components in DI system)

• Tested for cold gas (Nitrogen) and hot gas reaction products from decomposition of 70% peroxide (232 °C)
Hot Gas Position Tracking Results

Desired (blue) and Actual (green) Spool Angle: 5 Hz

Desired (blue) and Actual (green) Spool Angle: 10 Hz

Desired (blue) and Actual (green) Spool Angle: 15 Hz

Closed Loop Response of Hot Gas Valve
Hot Gas Three-Way Spool Valve
Proportional Injector Direct Injection Actuator Prototype

- Proportional Injectors
- Fuel Line
- Actuator
- Pressure Sensors
- Hot-Gas 3-way Spool Valve
Summary

Designed rotary actuated servovalves for both 3-way and 4-way configurations

Incorporated thermal isolation between the valve and actuator in order to accommodate the reaction products of 70% hydrogen peroxide (232°C)

Fabricated a prototype of the 3-way hot gas rotary actuated servovalve

Experimentally demonstrated the performance of the valve with both cold and hot gas (232°C)

-- Provides control of flow coefficient $C_v$ from 0 to 0.45
-- Exhibits position tracking bandwidth of 30 Hz for 50% of full scale motion
4-way Hot Gas Servovalve Requirements

- Proportional 4-way operation
- Operating pressure of 2 MPa (300 psi)
- Gas temperature of 232°C (450°F)
- Flow coefficient ~0.25
- Bandwidth > 25 Hz
- Fraction of volume, weight, and power consumption relative to motor/gearhead with equivalent cylinder output

CIM custom valve
(300 psi, 450°F, $C_v = 0.25$, 28 g)

Enfield Technologies
Model LS-V05
(120 psi, 115°F, $C_v = 0.13$, 108 g)

Enfield Technologies
Model LS-V15
(120 psi, 115°F, $C_v = 0.35$, 337 g)

FESTO
Model 154200 MPYE-5-M5-010-B
(120 psi, 115°F, $C_v = 0.1$, 297 g)
4-way Hot Gas Servo Valve Design

Valve Design
- Seal provided by ~1 micron spool/sleeve clearance
- 3 mm spool diameter
- Pressure-balanced spool minimizes frictional loads
- PEEK shaft/housing couplings provide thermal isolation
- Custom gearhead
  - Two stage cable transmission (9:1)
  - Adds 5 mm length (compared to 19 mm)
  - Estimated max temp of 160°C
  - Increases thermal isolation of interface
  - No backlash
4-way Hot Gas Servovalve Performance

Frequency Response of Servovalve

Mass Flow Rate for \( P_s = 2\text{MPa} \)

Measured Flow Characteristics
Short Course on Chemofluidics – Part 6

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Modeling and Control of a Monopropellant-Based Pneumatic Actuation System

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Taipei, Taiwan
Motivation

Problem
Conventional means of robotic power supply and actuation are not able to provide the energy density or power density required for a self-powered human-scale robot.

General Objective
Develop power supply and actuation system for human-scale robots that is as controllable as a battery/DC motor combination with order-of-magnitude greater energy and power density.

Approach
Utilize liquid monopropellants as gas generant for hot gas actuators.

Honda P3 humanoid robot
• Battery-powered DC motor actuation
• 130 kg (285 lb) total weight
• 30 kg (66 lb) of batteries
• 25 minutes
• No external work
• Max walk 1.6 mph
Producing Energy with a Monopropellant

\[
\text{propellant} + \text{catalyst} = \text{reaction products} + \text{heat}
\]
Tracking Control Demonstration

- 25 pound load
- 70% H₂O₂
- 220 psig
- Non model-based control
Objective: Model-Based Control

Modeling
First principles energetic approach
Analyze energetics within the catalyst pack
Analyze energetics within the reservoir tank

Control
- Track or maintain the pressure in the reservoir tank
- Control input is an on/off liquid valve
- Approach: direct evaluation of Lyapunov candidates
Modeling: Catalyst Pack

Energetic Modeling:

\[ \dot{U}_{cat} = H_{cat} + \dot{Q}_{cat} - \dot{W}_{cat} \]  
(energy storage rate = flux rate)

\[ \dot{U}_{cat} = 0 \quad \dot{W}_{cat} = 0 \]  
(no significant energy storage, no work done)

\[ \dot{H}_{cat} = P_s \dot{Q} - \dot{m}_{cat} c_p T_{ADT} \]  
(net enthalpy rate)

\[ \dot{Q}_{cat} = \dot{Q}_r \]  
(decomposition is dominant heat flux rate)

\[ \tau_r \ddot{Q}_r + \dot{Q}_r = km_{in} \]  
(1st order dynamic associated with heat released by catalytic decomposition)

\[ \dot{m}_{in} = \rho_l \dot{Q} = \rho_l c A \sqrt{P_s - P} \]  
(hydraulic mass flow)
Modeling: Catalyst Pack

\[ \dot{U}_{\text{cat}} = \dot{H}_{\text{cat}} + \dot{Q}_{\text{cat}} - \dot{W}_{\text{cat}} \quad (\text{energy storage rate} = \text{flux rate}) \]

\[ \dot{U}_{\text{cat}} = 0 \quad \dot{W}_{\text{cat}} = 0 \quad (\text{no significant energy storage, no work done}) \]

\[ \dot{H}_{\text{cat}} = P_s \tilde{Q} - m_{\text{cat}} c_p T_{ADT} \quad (\text{net enthalpy rate}) \]

\( P_s \tilde{Q} \)  Hydraulic flow power associated with the supply pressure of the monopropellant and the volumetric flow rate of liquid monopropellant (negligible)

\( m_{\text{cat}} c_p T_{ADT} \)  Pneumatic (i.e. compressible gas) flow power (dominant)

For 70% \( \text{H}_2\text{O}_2 \) in steady-state operation with a 4.9 MPa (700 psig) supply pressure, the ratio of \( m_{\text{cat}} c_p T_{ADT} \) to \( P_s \tilde{Q} \) is about 220.

Therefore: \( P_s \tilde{Q} = 0 \)
Modeling: Reservoir Chamber

Energetic Modeling:

\[
\dot{U}_{ch} = \dot{H}_{ch} + \dot{Q}_{ch} - \dot{W}_{ch} \quad \text{(energy storage rate = flux rate)}
\]

\[
\dot{U}_{ch} = \frac{1}{\gamma - 1} \left( \dot{P}V - P \dot{V} \right) \quad \text{(energy storage rate)}
\]

\[
\dot{H}_{ch} = \dot{m}_{ch} c_p T_{ADT} \quad \text{(enthalpy rate)}
\]

\[
\dot{m}_{ch} = \dot{m}_{cat} \quad \text{(continuity with catalyst pack CV)}
\]

\[
\dot{Q}_{ch} = -\frac{1}{R_h} (T - T_w) \quad \text{(heat loss through walls)}
\]

\[
\tau_h \dot{T}_w + T_w = T \quad \text{(heat storage in wall)}
\]
Modeling: Experimental Validation

![Graph showing pressure (kPa) over time for a 10 ms valve pulse.](Image)
Modeling: Experimental Validation

20 ms valve pulse

Pressures (kPa):
- 550
- 500
- 450
- 400
- 350
- 300
- 250
- 200
- 150

Time (s):
- 0.9
- 1
- 1.1
- 1.2
- 1.3
- 1.4
Modeling: Experimental Validation

30 ms valve pulse

Pressure (kPa) vs. Time (sec)
Control

The control of the catalyst pack / chamber system is unconventional in that the control input to the system (the binary monopropellant valve) is non-proportional in nature.

Approach: evaluate the effect of each discrete control choice (on or off) and select the choice either decreases the Lyapunov function (or increases it the least) at each time step.

Model can be written in the form:

\[
\dot{P} = f(x) + b(x)u
\]

\[
f(x) = -\frac{(\gamma - 1)}{\tau_ r V} \dot{Q}_ r - \frac{(\gamma - 1)}{V \tau_ h R_ h} (\dot{T} - T + T_ w)
\]

\[
b(x) = \frac{(\gamma - 1)}{\tau_ r V} k \rho_ t c A \sqrt{P_ s - P}
\]

Lyapunov function:

\[
V = \frac{1}{2} s^2
\]

\[
s = \dot{e} + 2\lambda e + \lambda^2 \int e dt
\]

\[
\dot{V} = ss \leq 0
\]
Switching Control Law

\[ V = \frac{1}{2} s^2 \]

\[ s = \dot{e} + 2\lambda e + \lambda^2 \int e dt \]

\[ \dot{V} = ss_s \leq 0 \quad \text{Drive } s \text{ to zero to enforce desired error dynamic.} \]

Evaluate the derivative of the Lyapunov function for each control choice (on or off):

\[ u_1 = 0 \quad \Rightarrow \quad \dot{s}_1 = f(x) - \ddot{P}_d + 2\lambda \dot{e} + \lambda^2 e \quad \Rightarrow \quad \dot{V}_1 = s \dot{s}_1 \]

\[ u_2 = 1 \quad \Rightarrow \quad \dot{s}_2 = f(x) + b(x) - \ddot{P}_d + 2\lambda \dot{e} + \lambda^2 e \quad \Rightarrow \quad \dot{V}_2 = s \dot{s}_2 \]

Pick the \( u \) with the correspondingly lowest \( \dot{V} \)
Control: Experimental Validation

0.0625 Hz (1/16 Hz) Trajectory

Pressure (kPa)
Control: Experimental Validation

0.25 Hz (1/4 Hz) Trajectory

Pressure (kPa)

0 0.2 0.4 0.6 0.8 1
0 0.25 0.5 0.75 1
Control: Experimental Validation

0.8 Hz Trajectory

Pressure (kPa)
Conclusions

A first-principles model based on the energetics of the catalyst pack / chamber system was presented.

Experimental results were shown validating the model.

A model-based control approach was presented to track an increasing desired pressure profile using an on/off liquid valve.

Experimental results were shown validating the model-based controller.

The model and control approach may include the effects of a variable volume chamber and are thus suitable for the control of a “direct injection” actuation system as shown below:
Short Course on
Chemofluidics – Part 7

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Force and Position Control of a Solenoid Injected Monopropellant Actuator

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Motivation

Conventional actuation technology does not possess the energy and power densities required for a human scale self-powered robot to perform external work for extended periods of time.

Work is motivated by the need to develop an actuation system with much higher energy and power densities than currently available.

Liquid chemical fuels have significantly greater energy densities than batteries. Specifically, system utilizes a monopropellant and catalyst pack as a gas generator for a pneumatic actuator.

Honda P3 humanoid robot
- Battery-powered DC motor actuation
- 130 kg (285 lb) total weight
- 30 kg (66 lb) of batteries
- 25 minutes
- No external work
- Max walk 1.6 mph
Producing Energy With a Monopropellant

\[ \text{propellant} + \text{catalyst} = \text{reaction products} + \text{heat} \]

- Hydrogen peroxide decomposes into a hot gas which can be used to power a pneumatic actuator
Solenoid Injector DI System

- Inert gas charge
- Fuel tank
- Monopropellant
- Fuel line
- Catalyst pack
- Exhaust valve (gas)
- Chamber A
- Chamber B
- Inlet valve (liquid)
- Actuator
Direct Injection Actuator Prototype

- Fuel Tank
- Fuel Valve
- Hot Gas Spool Valve
- Catalyst Pack
- Actuator
- Pressure Sensor
Actuator Modeling

• Applying the first law of thermodynamics, assuming an ideal gas undergoing an adiabatic process, the resulting pressure dynamics are given by

\[
\dot{P}_{(A,B)} = \gamma \frac{R}{V_{(A,B)}} (\dot{m}_{in(A,B)} T_{in(A,B)} - \dot{m}_{out(A,B)} T_{(A,B)}) - \gamma \frac{\dot{V}_{(A,B)}}{V_{(A,B)}} P_{(A,B)}
\]

• Outlet mass flow rate same as standard pneumatic systems, assumes isentropic flow of an ideal gas through a converging nozzle

\[
\dot{m}_{out(A,B)} = \begin{cases} 
\sqrt{\frac{\gamma}{RT} \left( \frac{2}{\gamma + 1} \right)^{(\gamma+1)/(\gamma-1)}} C_{f,ex} P_{(A,B)} A_{v,ex} & \frac{P_{atm}}{P_{(A,B)}} < C_r \\
\sqrt{\frac{2\gamma}{RT(\gamma-1)}} \left( 1 - \left( \frac{P_{atm}}{P_{(A,B)}} \right)^{(\gamma-1)/\gamma} \right)^{\gamma/\gamma} C_{f,ex} P_{(A,B)} A_{v,ex} & \frac{P_{atm}}{P_{(A,B)}} > C_r
\end{cases}
\]
Actuator Modeling

- Applying energy balance to the catalyst pack, assuming constant volume and small rate of change of internal energy in catalyst pack, then all heat produced by the reaction results in enthalpy at the catalyst pack outlet as described by

\[
\dot{Q}_{\text{cat}} = \dot{H}_{\text{cat}} = \dot{m}_{\text{cat}} c_p T_{\text{ADT}}
\]

- Assuming no heat lost through the walls of the catalyst pack, the heat released in the catalyst pack has been experimentally shown to be described by

\[
\tau_c \dot{Q}_{\text{cat}} + \dot{Q}_{\text{cat}} = k \dot{m}_{\text{fuel}}
\]

\[
\dot{m}_{\text{fuel}} = \rho_l \dot{Q} = \rho_l c_A v \sqrt{P_s - P}
\]

- Combining the above equations and the fact that \( \dot{m}_{\text{cat}} = \dot{m}_{\text{in}} \)

\[
\tau_c \ddot{m}_{\text{in}(A,B)} + \dot{m}_{\text{in}(A,B)} = \frac{k \rho_l C_{f,in} A_{v,in(A,B)} \sqrt{P_s - P_{(A,B)}}}{c_p T_{\text{ADT}}}
\]
DI Controller Components

- Control of the direct injection system separated into three components

  - First component: a predictive controller for the solenoid-operated injection valves that allow them to charge each chamber to a desired pressure.

  - Second component: designed a hot gas exhaust valve and a controller for the exhaust valve so the actuator can depressurize.

  - Third component: a supervisory controller that determines the desired pressurization and depressurization rates of each chamber and coordinates the operation of the 2 injection valves and the single exhaust valve to track a desired actuator force or position.
Solenoid Injection Valve Control

Problem:
- Need to proportionally control an output quantity via a binary on/off control effort subject to a time delay.

Approaches:
- PWM-type switching controllers
  - Provide a convenient means of obtaining proportionality of effort from a binary source
  - Require high-bandwidth valves and extensive, unnecessary switching
  - System dynamics in this case are too fast relative to valve dynamics to use PWM
- Direct-type switching controllers
  - Typically use model knowledge to minimize a Lyapunov function
  - Require prediction in the presence of time delay
- Predictive-type switching controllers
  - Uses model knowledge to project effects of current valve commands on future output
  - Replaces proportionality in amplitude with *low-bandwidth* proportionality in time
  - Valve dynamics can be on the same order as system dynamics
Proposed Predictive Control Approach

Uses a model of the system to predict the value of a Lyapunov function at a future time horizon for each (next time-step) control output candidate and chooses the next control output that corresponds to the smallest predicted future Lyapunov function.

We assume linear system dynamics in order to incorporate closed-form solution to linear system dynamics.

Applicable to nonlinear systems, but would require a non-closed-form nonlinear solver (e.g., Numerical integration).
Predictive Control Methodology

• Given a linear time-invariant state space model with time delay $T_u$ in the input:

$$ \dot{x}(t) = Ax(t) + Bu(t - T_u) $$

• The predictive control law is given by

$$ u(t^+) = \begin{cases} 
0 & \text{if } \hat{V}_o(t + T_u + T_d) \leq \hat{V}_i(t + T_u + T_d) \\
1 & \text{else}
\end{cases} $$

• The predicted value of the Lyapunov function for each input candidate at the prediction horizon is given by

$$ \hat{V}_i (t + T_u + T_d) = \frac{1}{2} \left( d \frac{d}{dt} + \lambda \right)^{n-1} \left( \hat{x}_i (t + T_u + T_d) - x_d (t) \right)^2 $$
Predictive Control Methodology

• Predicted response projected $T_u + T_D$ into the future ($T_D$ is the prediction horizon) for each case $u = 0$ or $u = 1$ for the next switching period $T_s$:

$$\hat{x}(t + T_u + T_D) = e^{A(T_u + T_D)}x(t) + \int_{t}^{t+T_u} e^{A(T_u + T_D - \tau)}Bu(\tau - T_u)d\tau + \begin{cases} 0, & \text{for } u = 0 \\ \int_{t+T_u}^{t+T_u+T_s} e^{A(T_u + T_D - \tau)}Bd\tau, & \text{for } u = 1 \end{cases}$$

• Choosing the control law therefore amounts to choosing the next control input that results in the smallest predicted Lyapunov function at time $t+T_u+T_D$.

• The predictive control law above is evaluated every switching period $T_s$, so the controller continuously evaluates the predicted response for each possible input.
Linear Model of the Actuator

- In order to apply the predictive control approach, a linear model of the actuator pressurization dynamics must be obtained. The previously derived nonlinear pressure dynamics were linearized about an operating point

\[ \dot{P}_{(A,B)} = k_{1(A,B)} \dot{m}_{in(A,B)} - k_{2(A,B)} P_{(A,B)} \]

\[ k_{1(A,B)} = \gamma \frac{RT}{V_{(A,B)}} \]

\[ k_{2(A,B)} = \gamma \frac{\dot{V}_{(A,B)}}{V_{(A,B)}} \]

- \( k_1 \) and \( k_2 \) are slowly varying parameters relative to the pressure dynamics

- Inlet mass flow rate can linearly approximated as

\[ \tau_c \ddot{m}_{in(A,B)} + \dot{m}_{in(A,B)} = k_m u_{(A,B)} \]
Linear Model of the Actuator

- Predictive controller requires a state space representation of the linear model; the system states, system input, and the A and B matrices are given by:

\[
\mathbf{x}_{(A,B)} = \begin{bmatrix} \dot{m}_{in,(A,B)} \\ P_{(A,B)} \end{bmatrix}
\]

\[
u_{(A,B)} \in \{0,1\}
\]

\[
\mathbf{A}_{(A,B)} = \begin{bmatrix}
-\frac{1}{\tau_c} & 0 \\
\frac{1}{\tau_c} & -k_2(V_{(A,B)}, \dot{V}_{(A,B)}) \\
\end{bmatrix}
\]

\[
\mathbf{B}_{(A,B)} = \begin{bmatrix} \frac{k_m}{\tau_c} \\ 0 \end{bmatrix}
\]
DI Pressurization Control Experiments

Pressurization Tracking of a 0.25 Hz Sine Wave

Pressurization Tracking for a 1 Hz Sine Wave

Pressurization Tracking for a 2 Hz Sine Wave
Exhaust Valve Development

- A proportional controller that operates on the pressure error in each chamber commands the exhaust valve position.

- Valve spool position highly susceptible to frictional effects, closed-loop position controller required.
DI Pressurization/Depressurization Control Experiments

Pressure Tracking for 0.25 Hz Sine Wave

Pressure Tracking for 1 Hz Sine Wave

Pressure Tracking for 2 Hz Sine Wave

Pressure Tracking for 3 Hz Sine Wave
Supervisory Force Controller

Controller determines the respective pressurization/depressurization rates and the desired pressures in each chamber.

Coordinates the operation of the 2 binary injection valves and the single proportional exhaust valve to provide actuator force or position tracking.
Supervisory Force Controller with Average Pressure Loop

Switching Condition

Charge A if positive
Discharge A if negative

Charge B if positive
Discharge B if negative

Predictive controller injection valve A
Predictive controller injection valve B
Exhaust valve controller

Pneumatic actuator (plant)
DI Actuation System Tracking a 1 Hz Sinusoid
Force/Position Tracking Tests

Position Tracking, 1 Hz Sinusoid

Position Tracking, 0.25 Hz Square Wave

Corresponding Force Tracking

Corresponding Force Tracking
Tracking a Random Human Input

Position Tracking of an Arbitrary Human Input

Commanded Force vs. Actual Force

Time (Sec.)

Position (deg)

Force (lbs)

Commanded
Actual
Summary

- Implemented predictive model-based approach to control pressurization rates in a hot gas cylinder using solenoid injection valves
- Implemented control of a 3-way proportional exhaust valve in order to control depressurization rates
- Developed a supervisory control structure to determine desired pressurization/depressurization rates for each cylinder chamber and to coordinate the activity of the two injection valves and single exhaust valve to obtain desired output force tracking of the actuator
- Experimentally demonstrated pressure tracking of a single chamber (using predictive pressurization control and proportional exhaust control) and force tracking of the actuator (under guidance of the supervisory control structure)
Short Course on Chemofluidics – Part 8

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Servo Control and Energetic Characterization of a Proportionally Injected Liquid-Fueled Actuator for Power-Autonomous Mobile Robots

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Proportional Injector DI System

- Inert gas charge
- Fuel tank monopropellant
- Fuel line
- Catalyst pack
- Exhaust valve (gas)
- Inlet valve (liquid)
- Actuator
Injection Valve Design Considerations

- **Desired Performance Characteristics**
  - Operation at pressures of 2.0 MPa (300 psi)
  - High-resolution, high-bandwidth flow control
  - Maximum flow rates of ~1 mL/sec

- **Accommodate high temperature catalyst pack (232°C)**
  - Design must incorporate sufficient thermal isolation between the actuator and valve/catalyst pack

- **Minimal dead volume between injection orifice and catalyst pack**
  - Serves to minimize transport delay in the propellant decomposition dynamic
Proportional Injection Valve Design

- Incorporates concepts from a variety of common valve designs
- Can be described as a hybrid poppet-needle-diaphragm valve
- Utilizes a DC motor/gearhead/encoder package for valve actuation
- Provides requisite flow resolution with a closed-loop bandwidth on the order of 10 Hz
Sealing the Proportional Valve

- Initial prototypes used the gearbox shaft as valve spool and incorporated shaft seals for propellant sealing
  - High seal friction required increased actuator power
  - Low seal friction allowed leakage and subsequent gearhead corrosion
- Current design utilizes a diaphragm to isolate the servomotor and pressurized fuel
  - Viton diaphragm of 0.74 mm (0.029 in) thickness
  - Valve cap/body provides 0.18 mm (0.007 in) of diaphragm compression to seal against 2.0 MPa (300 psi) working fluids
Valve Actuation Through Diaphragm

- Lead screw assembly controls the position of valve poppet

- Actuation through the diaphragm is unidirectional:
  - motor can actuate the poppet to close the valve
  - relies on restoring forces to (e.g. fluidic or mechanical) to open the poppet

- Initial designs used a flat-bottomed bronze foot to seal the diaphragm directly against the inlet orifice:
  - resulting flow resolution was poor (valve essentially behaved as a low-bandwidth discrete valve)
  - sealing at inlet orifice induced significant dead volume (and corresponding transport delay)
Obtaining High-resolution Proportional Flow Control

- Final design incorporates a tapered poppet sealed at the outlet orifice of valve
- Taper provides larger flow resolution by decreasing sensitivity of annular orifice area on as a function of motor rotation
- Wave springs pretension the poppet against leadscrew/diaphragm
  - prevents jamming/sticking of the tapered poppet within the orifice
  - provides sealing at outlet orifice, thereby minimizing dead volume between valve and catalyst pack
Proportional Injection Valve Characteristics

- Maximum flow rate of 9 mL/sec
- Closed-loop bandwidth of 10 Hz
- Power production package geometry:
  -- Length – 18 cm
  -- Max. Diameter – 2.5 cm
PIDI Actuator Package

- Hot-gas cylinder
- Injection valves
- Pressure sensors
- 3-way exhaust valve
Modeling of the PIDI Actuator

Force Output of the Hot Gas Actuator:

\[ F_{act} = P_A A_A - P_B A_B + P_{atm} A_r \]

Compressible Gas Dynamics:

\[ \dot{P} = \frac{RT}{V} (\dot{m}_{in} - \dot{m}_{out}) - \frac{\dot{V}}{V} P \]

Injection Dynamics (neglecting reaction dynamics):

\[ \dot{m}_{in} = A_{in} c \sqrt{2 \rho_L (P_s - P)} \]

Exhaust Flow Relations:

\[ \dot{m}_{out} = \Psi(P) u_{out} \]

\[ \Psi(P) = \begin{cases} \sqrt{\frac{\gamma}{R} \left( \frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma}}} \frac{C_f P}{\sqrt{T_{ADT}}} & \text{if } \frac{P_{atm}}{P} \leq C_r \text{ (choked)} \\ \frac{2\gamma}{R(\gamma - 1)} \frac{C_f P}{\sqrt{T_{ADT}}} \left( \frac{P_{atm}}{P} \right)^{\gamma/2} \left(1 - \left( \frac{P_{atm}}{P} \right)^{(\gamma - 1)/\gamma} \right) & \text{otherwise (unchoked)} \end{cases} \]
PIDI Force (Pressure) Control

\[
P_{A,d} = \frac{1}{A_A} \left( F_d + P_B A_B + P_{am} A_r \right) \quad \text{for} \quad F_d \geq 0
\]
\[
\Rightarrow \quad \dot{m}_{A,d} = \begin{cases} 
K_p \left( P_{A,d} - P_A \right) & \text{for} \quad F_d \geq 0 \\
0 & \text{otherwise}
\end{cases}
\]

\[
P_{B,d} = \frac{1}{A_B} \left( -F_d + P_A A_A - P_{am} A_r \right) \quad \text{for} \quad F_d < 0
\]
\[
\Rightarrow \quad \dot{m}_{B,d} = \begin{cases} 
K_p \left( P_{B,d} - P_B \right) & \text{for} \quad F_d < 0 \\
0 & \text{otherwise}
\end{cases}
\]

\[
u_{in,A} = A_{in,A} = \begin{cases} 
\dot{m}_{A,d} & \text{for} \quad \dot{m}_{A,d} \geq 0 \\
0 & \text{otherwise}
\end{cases}
\]

\[
u_{in,B} = A_{in,B} = \begin{cases} 
\dot{m}_{B,d} & \text{for} \quad \dot{m}_{B,d} \geq 0 \\
0 & \text{otherwise}
\end{cases}
\]

\[
u_{out,A} = \frac{-\dot{m}_{A,d}}{\Psi(P_A)} \quad \text{for} \quad \dot{m}_{A,d} < 0
\]

\[
u_{out} = \begin{cases} 
0 & \text{for} \quad \dot{m}_{A,d}, \dot{m}_{B,d} \geq 0 \\
-\dot{m}_{B,d} & \text{for} \quad \dot{m}_{B,d} < 0
\end{cases}
\]
PIDI Servo Control

Outer Position Control Loop:

- Proportional-Derivative Control with Feedforward Gravity Compensation

\[ \tau = K_{p,arm} (\theta_d - \theta) + K_{d,arm} (\dot{\theta}_d - \dot{\theta}) + m_a g l_1 \sin \theta_d \]

Desired actuator force obtained via inverse Jacobian:

\[ F_d = \left( \frac{\sqrt{d_1^2 + d_2^2 - 2d_1 d_2 \cos(\theta + \alpha)}}{d_1 d_2 \sin(\theta + \alpha)} \right) \tau \]
Hot Gas Position Tracking

Results

Arm Tracking for 1 Hz Motion Command

Force Tracking for 1 Hz Motion Command
Hot Gas Position Tracking Results (II)

Arm Position Tracking for 0.25 Hz Step Command

Force Tracking for 0.25 Hz Step Command
Energetic conversion efficiency characterized for sinusoidal 1 Hz motion using a 25 lb load

\[ P(t) = |\tau \dot{\theta}| \]

\[ P_{avg} = \frac{\int_{t_1}^{t_2} P(t) dt}{t_2 - t_1} \]

\[ \eta = \frac{P_{avg}(t_2 - t_1)}{\hat{e}_s m_{H_2O_2}} \]

For 70% H₂O₂ (\( \hat{e}_s = 0.4 \text{ MJ/kg} \)), the average conversion efficiency is measured to be:

\[ \eta = 45\% \]
Power Density

\[ P_{\text{peak}} = 365 \text{ W} \]
\[ m_{\text{PIDI}} = 0.95 \text{ kg} \]
\[ p_a = 384 \text{ W/kg} \]
Actuation Potential

Energetic Figure of Merit:

\[ A_p = e_s \eta \rho_a \]

Accounting for the mass of a 2 kg composite fuel tank:

\[ e_s = \frac{m_{fuel} \hat{e}_s}{m_{fuel} + m_{tank}} = 0.35 \text{ MJ/kg} \]

\[ A_p = (0.35)(45\%)(384) = 60.5 \text{ kJ·kW/kg}^2 \]

In comparison, for a neodymium-based servo motor with harmonic drive gearhead, powered with Ni-Zn batteries:

\[ A_p = 4.8 \text{ kJ·kW/kg}^2 \]
Summary

Developed prototype proportional poppet-needle-diaphragm valve with high resolution, a 10-Hz tracking bandwidth, and maximum flow rate of 9 mL/sec

Developed and implemented proportional pressure control loop for PIDI actuator force tracking and outer loop servo control for arm position tracking

Experimentally characterized the energetic performance of the PIDI actuator:

- Energetic conversion efficiency – 45%
- Power density – 385 W/kg
- Actuation Potential – 60.5 kJ·kW/kg²